

Factoring Polynomials $ax^2 + bx + c$

(Equation must be in highest power order unless 2 variables are squared than 1 on each end) (solving must = 0)

1. GCF & Leading Negative

- a. Example: $-3x^3 + 12x^2 - 6x = 0$ factor out the $-3x$ so, $-3x(x^2 - 4x + 2) = 0$

2. Special Conditions

- a. **Difference of two squares** (1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 225...)
i. General equation: $a^2 - b^2 = (a - b)(a + b)$ Ex. $4x^2 - 25 = (2x - 5)(2x + 5)$
ii. Ex. $3x^2 = 75$, (1st set = 0, 2nd divide both sides by 3 so, $x^2 - 25 = 0$) $(x+5)(x-5)$
iii. Ex. $3(x+2)^2 = 108$; $(x+2)^2 = 36$; $(x+2)^2 - 36 = 0$; $((x+2)+6)((x+2)-6) = 0$
- b. **Sum or Difference of two cubes** (1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331...)
i. General equation: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ or $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
ii. Example: $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ (1st cube root each term (), then pattern)
- c. **Perfect Square Trinomial**: (both 1st and 3rd term must be perfect squares)
i. General Equation: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$
ii. Example: $4x^2 + 12x + 9 = (2x + 3)^2$

3. Without leading co-efficient

- a. Find Factors of the last term (list or use calc.) that add or subtract to equal the middle term.
b. Write out two sets of () with the variable as 1st term and factor as second term.
i. Ex. : $x^2 - 12x + 27 = 0$ Factors of 27 : (1 x 27 and 3 x 9, since $-3 \times -9 = -12$ use that)
ii. $(x - 3)(x - 9) = 0$

4. With a leading co-efficient AC Or Star: $\left(\frac{ax}{f}\right) \left(\frac{ax}{f}\right)$ Reduced, f= factors of ac that +or- to = b

- a. Find factors of $a * c$ (1st and last term) that add or subtract to = b (the middle term)
b. Re-write your trinomial (3 terms) as 4 terms replacing b with the above factors
c. Group by two's with a + btw () **(If 4 Terms Start Here)**
d. GCF & leading negative each separate group () + () (goal is for new () to match)
e. Write the outside factors together in one () times the matching ()
i. Ex. $2x^2 - 5x - 12 = 0$ a. ($2 * -12 = -24$; factors that equal -5 are, 3 & -8)
ii. $2x^2 + 3x - 8x - 12 = 0$ b. (write as 4 terms replacing b with factor and variable)
iii. $(2x^2 + 3x) + (-8x - 12)$ c. (group by 2's w/ + btw)
iv. $x(2x + 3) - 4(2x + 3)$ d. (GCF & Leading Negative)
v. $(x - 4)(2x + 3) = 0$ e. (put outside factors together () * inside ())

5. Completing the Square (or can use vertex method: $h = -b/2a$, $k = f(h)$)

- a. Isolate the x's on one side of = (move C rt), have no leading co-efficient (factor out if need to)
b. Create a perfect square by formula $(b/2)^2$ added as term on both sides of = (on rt side if an a mult)
c. Complete the square factoring as above 2c (perfect square tri $(a^2 \pm ab + b^2) = (a \pm b)^2$)
d. Solve either by square root both sides or other
i. Ex. $3x^2 - 8x + 1 = 0$, $3x^2 - 8x = -1$; $3(x^2 - \frac{8}{3}x) = -1$
ii. $3(x^2 - \frac{8}{3}x + (\frac{-8}{3}/2)^2) = -1 + 3((\frac{-8}{3})/2)^2$
iii. $3(x^2 - \frac{8}{3}x + \frac{16}{9}) = \frac{13}{3}$

6. $(x^2 - \frac{8}{3}x + \frac{16}{9}) = \frac{13}{3}$; $(x - \frac{4}{3})^2 = \frac{13}{3}$; $\sqrt{(x - \frac{4}{3})^2} = \pm\sqrt{\frac{13}{3}}$; $x - \frac{4}{3} = \pm\sqrt{\frac{13}{3}}$; $x = \frac{4}{3} \pm \frac{\sqrt{39}}{3}$

7. **Quadratic equation:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Must be in Standard form 1st than pull out the a, b, c (no x's)